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Applications and Interpretation Formulae Sheet (Standard Level and Higher Level)

Pre-Requisites

Area of Triangle	$\frac{1}{2}$ x base x height
Area of Parallelogram	base x height
Area of Rectangle	$length \times width$
Area of Trapezoid	$\frac{1}{2}$ (sum of parallel sides) x height
Circumference & Area: Circle	$c = 2\pi r,\ A = \pi r^2$
Cuboid Surface area	$SA = 2xy + 2xz + 2yz$ Where $x, y, and\ z$ are side lengths
Cuboid Volume	$V = xyz$ where $x, y, and\ z$ are side lengths
Cylinder Surface Area	$SA = 2\pi rh + 2\pi r^2$ Note: Curved part: $2\pi rh$
Cylinder Volume	$V = \pi r^2 h$
Prism Volume	V =Area of cross section x height
Distance between 2 points $(x_1, y_1), (x_2, y_2)$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Coordinates of midpoint of $(x_1, y_1), (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Solutions of a Quadratic Equation $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$

Topic 1: Number and Algebra

Arithmetic sequence: nth term	$u_n = a + (n-1)d$ where a =first term, d= common diff
Arithmetic sequence: sum of n terms	$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$ where a =first term, d= common diff, l=last term
Geometric sequence: nth term	$u_n = ar^{n-1}$ where a =first term, r= common ratio
Geometric sequence: sum of n terms	$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}, r \neq 1$ where a =first term, r= common ratio
Geometric sequence: Sum to infinity	$S_\infty = \frac{a}{1-r},\ \lvert r \rvert < 1$ where a =first term, r= common ratio
Compound Interest	$FV = PV\left(1 + \frac{r}{100\ k}\right)^{kt}$ FV=future value PV=present value t=no. of years r=nominal annual interest rate k =no. of compounding periods per year
Exponential & Logarithms	$\log_a b = c \Leftrightarrow a^c = b,\ a, b, > 0, a \neq 1$
Exponential & Logarithms Rules	• $c \log_a b \Leftrightarrow \log_a b^c$ • $\log_a b + \log_a c \Leftrightarrow \log_a bc$ • $\log_a b - \log_a c \Leftrightarrow \log_a \frac{b}{c}$ • $\log_a b \Leftrightarrow \frac{\log_c b}{\log_c a}$
Percentage Error	$\varepsilon = \left\lvert \frac{v_A - v_E}{v_e} \right\rvert \times 100\%$ v_A =approximate value, v_E =exact value
Quadratic Function: Discriminant	$\Delta = b^2 - 4ac$ • > 0 (2 real distinct roots) • $= 0$ (2real repeated/double roots) • < 0 (no real roots)
Complex Numbers: Cartesian Form	$z = a + bi$
Complex Numbers: Modulus/Argument Form	$z = r(\cos\theta + i \sin\theta) = r\ cis\ \theta$
Complex Numbers: Eulers Form	$z = re^{i\theta}$
Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = \lvert A \rvert = ad - bc$
Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
Power formula for a matrix	$M^n = PD^nP^{n-1}$, where **P** is the matrix of eigenvectors and **D** is the diagonal matrix of eigenvalues

Topic 2: Functions

Straight Line: Equation (gradient means slope) Parallel⟹ same slope Perpendicular⟹ "flip fraction and change the sign" (slopes multiply to make −1)	• Slope intercept $y = mx + c$ • General $ax + by + d = 0$ • Point slope form $y - y_1 = m(x - x_1)$
Straight Line: Gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Quadratic Function:Solutions to $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
Quadratic Function: Axis of Symmetry	$f(x) = x^2 + bx + c \Rightarrow x = -\frac{b}{2a}$
Inverse	Replace $f(x)$ with y, swap x & y, solve for y
Composite	$fg(x)$ means plug $g(x)$ into $f(x)$
Transformations: $af(bx + c) + d$ "anything in a bracket does the opposite"	a=vertical stretch sf a, b=horizontal stretch sf $\frac{1}{b}$, c=translation c units x direc, d=translation d units in y direction $f(-x)$=reflc in $y\ axis$, $-f(x)$=reflc in $y\ axis$

Linear: $y = mx + c$ Domain: x∈ ℝ Range: y∈ ℝ **Quadratic:** $y = \pm a(bx + c)^2 + d$ Domain: x∈ ℝ Range: $y \geq d$ if min, $y \leq d$ if max **Exponential:** $y = ae^{bx+c} + d$ Domain: x∈ ℝ (Hint: power of exp can be anything, so no restriction) Range: $y > d$ if a > 0, $y < d$ if a < 0 **Logarithm:** $y = a\ln(bx + c)$+d Domain: $x > -\frac{c}{b}$ Range: y ∈ ℝ Asymptote: $x = -\frac{c}{b}$ **Root:** $a\sqrt{bx + c} + d$: Domain: $x \geq -\frac{c}{b}$ Range: $y \geq d$ if $a > 0$ and $y \leq d$ if a<0 **Rational:** $\frac{ax+b}{cx+d} + d$ Domain: x∈ ℝ, $x \neq -\frac{d}{c}$ Range: y∈ ℝ, $y \neq \frac{a}{c} + d$ Asymptotes: $x = -\frac{d}{c}, y = \frac{a}{c} + d$	**Trigonometry:** $y = a\sin(bx + c)$+d $y = a\cos(bx + c)$+d Domain: $x \in$ ℝ Range: $-a + d \leq y \leq a + d$ Note: If asked to find values of a,b,c,d a = amplitude=$\frac{\max y - \min y}{2}$ $b = \frac{2\pi}{period}$ or $\frac{360}{period}$ d = principal axis=$\frac{\max y + \min y}{2}$ c =phase shift (plug in point to find) **Logarithm:** $y = a\ln(bx + c)$+d Asymptote: $x = -\frac{c}{b}$

Logistic Function	$f(x) = \frac{L}{1 + Ce^{-kx}},\ L, k, C > 0$

Topic 3: Geometry and Trigonometry

Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Coordinates of midpoint of (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Cone Surface Area	$SA = \pi rl + \pi r^2$ Note: Curved part: πrl where l is slant length
Cone Volume	$V = \frac{1}{3}\pi r^2 h$
Sphere Surface Area	$SA = 4\pi r^2$ Note: Hemisphere= $2\pi r^2 + \pi r^2 = 3\pi r^2$
Sphere Volume	$v = \frac{4}{3}\pi r^3$ Note: Hemisphere= $\frac{2}{3}\pi r^3$
Pyramid Volume	$V = \frac{1}{3} \times base\ area \times h$
Sine Rule	Finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Cosine Rule	inding a side: $a^2 = b^2 + c^2 - 2bc\cos A$ Finding an angle:$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$
Area of Triangle	$\frac{1}{2}ab\sin C$
Degrees to radians and vice versa	D to R: $\times \frac{\pi}{180}$ R to D: $\times \frac{180}{\pi}$
Length of an arc	$\frac{\theta}{360} \times 2\pi r$
Area of a Sector	$\frac{\theta}{360} \times \pi r^2$
Trig Identities	$\sin^2 x + \cos^2 x = 1$ $\tan x = \frac{\sin x}{\cos x}$
Transformation Matrices	• $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ Reflection in the line $y = (\tan\theta)x$ • $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ Horizontal stretch by scale factor k • $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ Vertical stretch by scale factor k • $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ Enlargement by scale factor k, centre (0,0) • $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ Anti-clockwise rotation of angle θ about the origin $(\theta > 0)$ • $\begin{pmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ clockwise rotation of angle θ about the origin $(\theta > 0)$
Vector Form	$\boldsymbol{ai} + \boldsymbol{bj} + \boldsymbol{ck} \equiv \begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Properties (addition/ subtraction, multiplication and scalar product)	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \pm \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a \pm d \\ b \pm e \\ c \pm f \end{pmatrix}$ $\lambda\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \lambda a \\ \lambda b \\ \lambda c \end{pmatrix}$ $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$
Magnitude of a vector	$\left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rvert = \sqrt{a^2 + b^2 + c^2}$
Unit Vector	Unit vector of $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{\sqrt{a^2 + b^2 + c^2}}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$
Angle Between 2 vectors	$\theta = \cos^{-1}\left(\frac{\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix}}{\left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\rvert}\right)$
Vector Equation of a line	$r = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda\begin{pmatrix} d \\ e \\ f \end{pmatrix}$
Parametric Form of a line	$x = a + \lambda d, y = b + \lambda e, z = c + \lambda f$
Scalar Product	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\rvert \cos\theta$ where, θ is the angle between $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$
Vector Product	$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} bf - ec \\ -(af - cd) \\ ae - bd \end{pmatrix}$ or $\left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\rvert = \left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\rvert \sin\theta$ where, θ is the angle between $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$
Area of a Parallelogram	$A = \left\lvert \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\rvert$ where, $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$ form 2 adjacent sides of a parallelogram

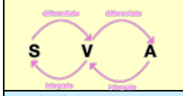
Topic 4: Statistics & Probability

Interquartile Range	IQR= $Q_3 - Q_1$
Mean	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ where $n = \sum_{i=1}^{k} f_i$
Probability of event A	$P(A) = \frac{n(A)}{n(U)}$
Complementary Events	P(A')+1−P(A)
Combined Events	P(A∪B)=P(A)+P(B)−P(A ∩ B)
Mutually Exclusive Events	P(A ∩ B) = 0
Conditional	P(A|B) = $\frac{P(A \cap B)}{P(B)}$
Independent Events	P(A ∩ B) = P(A)P(B)
Expected Value – discrete random variable	E(X)= $\sum xP(X = x)$
Binomial Distribution	$x \sim B(n, p)$ E(X)=Mean= np, Var(X)= $np(1-p)$

Topic 4: Statistics & Probability Continued

Linear Transformations of a random variable	$E(aX \pm b) = aE(X) \pm b$ $Var(aX \pm b) = a^2 Var(x)$
Linear Combinations of n independent random variables $X_1, X_2, \dots, X_n$	$E(a_1 X_1 \pm a_2 X_2 \pm \cdots \pm a_n X_n) = a_1 E(X_1) \pm a_2 E(X_2) \pm \cdots \pm a_n E(X_n)$ $Var(a_1 X_1 \pm a_2 X_2 \pm \dots \pm a_n X_n) = a_1^2 Var(X_1) + a_2^2 Var(X_2) + \cdots + a_n^2 Var(X_n)$
Unbiased estimate of a population variance	$s_{n-1}^2 = \frac{n}{n-1} s_n^2$
Poisson Distribution	$x \sim P_0(m)$ E(X)= m, Var(X)= m
Transition Matrices	$T^n s_0 = s_n$, where s_0 is the initial state

Topic 5: Calculus

Turning/Stationary Points (Max/Min)	Solve $\frac{dy}{dx} = 0$, If $\frac{d^2y}{dx^2} > 0$ min and $\frac{d^2y}{dx^2} < 0$ max
Points of Inflection	solve $\frac{d^2y}{dx^2} = 0$
Increasing/Decreasing	Increasing: solve $\frac{dy}{dx} > 0$, decreasing: solve $\frac{dy}{dx} < 0$
Convex/Concave	concave up: solve $\frac{d^2y}{dx^2} > 0$ concave down: solve $\frac{d^2y}{dx^2} < 0$
Chain Rule	$y = g(u), u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Product Rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Area between	curve & x axis: $\int_{x=a}^{x=b} y\,dx$ curve & y axis: $\int_{y=a}^{y=b} x\,dy$ (take + answer if neg) Between 2 curves: $\int_{x=a}^{x=b}$(top curve-bottom curve)dx Remember to split up if separate areas
Trapezoidal Rule	$\int_a^b y\,dx \approx \frac{1}{2}h[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})]$ Where $h = \frac{b-a}{n}$
Kinematics: 	Distance=$\int_{t_1}^{t_2} \lvert v(t) \rvert dt$, Displacement=$\int_{t_1}^{t_2} v(t)dt$ Velocity: $\int_{t_1}^{t_2} a(t)dt$ or $\frac{ds}{dt}$ Acceleration= $\frac{dv}{dt} = \frac{d^2s}{dt^2}$
Derivatives	• $x^n \Rightarrow nx^{n-1}$ • $(f(x))^n \Rightarrow n(f(x))^{n-1} f'(x)$ • $\ln(f(x)) \Rightarrow \frac{f'(x)}{f(x)}$ • $\sin f(x) \Rightarrow f'(x)\cos f(x)$ • $\cos f(x) \Rightarrow -f'(x)\sin f(x)$ • $e^{f(x)} \Rightarrow e^{f(x)} f'(x) = e^{f(x)} f'(x)$
Integrals	• $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ • $\int \frac{1}{x} dx = \ln\lvert x \rvert + c$ • $\int \sin x\,dx = -\cos x + c$ • $\int \cos x\,dx = \sin x + c$ • $\int \frac{1}{\cos^2 x} dx = \tan x + c$ • $\int e^x dx = e^x + c$
Volume of Revolution	About $x\ axis$ $V = \int_a^b \pi y^2 dx$ About y axis: $V = \int_a^b \pi x^2 dy$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n); x_{n+1} = x_n + h$ where h is a constant (step length)
Euler's Method for Coupled Systems	$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $x_{n+1} = x_n + h \times 2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ where h is a constant (step length)
Exact solution for coupled linear differential equations	$x = AAe^{\lambda_1 t} p_1 + Be^{\lambda_2 t} p_2$